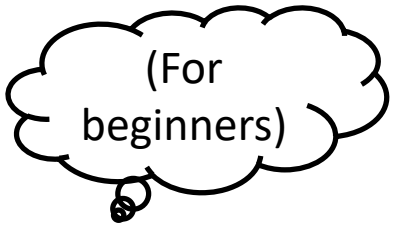
The maths behind Greenpower



90 minutes • 36Ah battery• 700W motor

The challenge: go as far as possible in 90 minutes.

So… two questions……

- How much energy have we got? [i.e. how big is the petrol tank?]
- How can we best use that energy?

So how much energy do we really have?

Two batteries, each 12V, and each rated at 36Ah. Put these in series and we get 24V but still only 36Ah.
But hang on – looking at the data sheet for the battery, that 36Ah is only if we discharge the battery over 20 hours, it drops to 32Ah over 10 hours, and we are only racing for 90 minutes…..

General Specifications

Specifications	REC10-12	REC12-12	REC14-12	REC…	REC26-12	REC36-12
Nominal voltage (V)	12	12	12	12	…	12
20-hr rate Capacity to 1.75VPC at 20°C (Ah)	10.00	12.00	13.00	22.00	26.00	36.00
10-hr rate Capacity to 1.75VPC at 20°C (Ah)	9.30	11.60	11.90	19.70	23.50	32.00
Dimensions:						

If we want to know what to expect over 90 minutes we have to use this formula

$$I^k t = K$$

This is **Peukert's law**, describing how battery discharge current affects runtime.

- I — constant discharge current, in amperes (A).
- k — Peukert exponent. For your battery ratings, we calculated $k \approx 1.205$.
- t — time until the battery reaches the specified cutoff voltage, in hours.
- K — a fitted constant for that battery and those test conditions. Using amperes and hours, yours is approximately **40.60**. It is not the battery's amp-hour capacity.

This gives us

90 minutes = 23.2 Ah

60 minutes = 21.6 Ah

So we now know how much energy we have – how do we use it best?

2 possible strategies…

Go like hell and hope for the best.



Trundle like Miss Daisy to make sure of lasting for 90 minutes.



….or is there something in between?

Remember Watts = Volts x Amps

23.2Ah x 24V = 557**Wh** of usable energy. Divide this by 1.5 and we get a maximum power of 371**W**

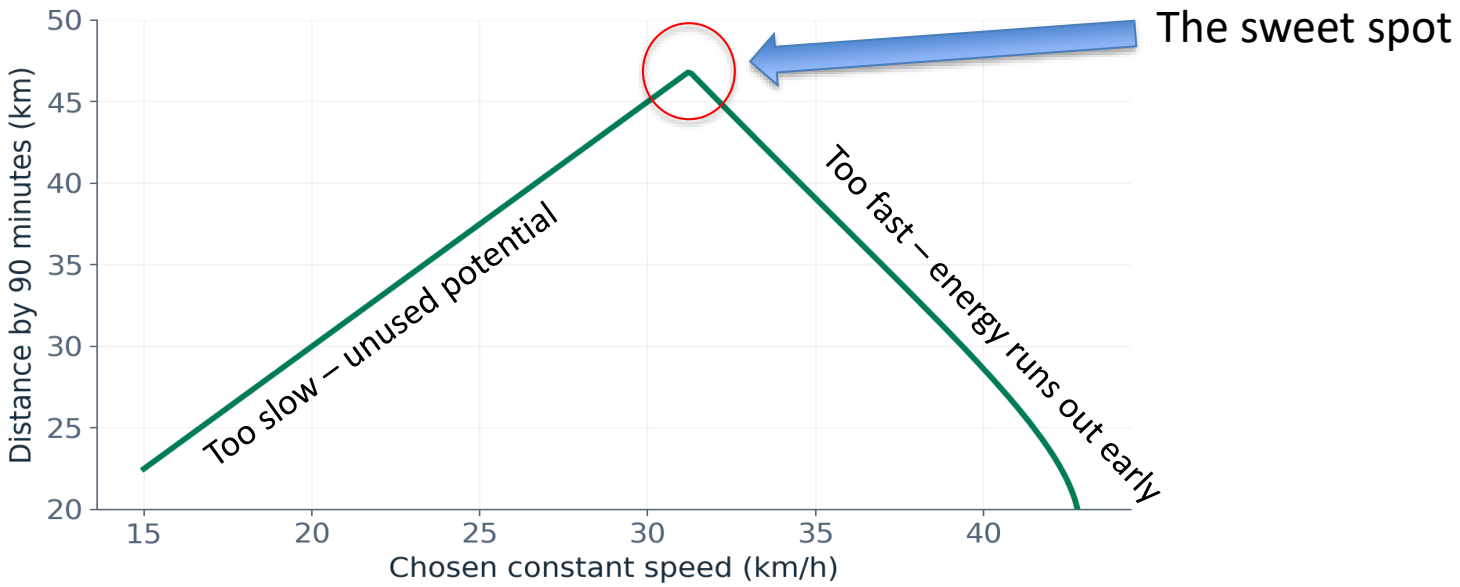
Divide this by 24V to get back to Amps and we get a maximum current of 15.5 A.

i.e. if we run our motor at **15.5A**, we should just run out of energy after **90** minutes.
[Or **21.6A** for the 60 minute F24+ races]

Assumptions:
- The data sheet is based on running the batteries down to 10.5 V, it is possible to go lower – even down to 9.6V but this only gets about 0.5% - 1% extra capacity and risks damaging the batteries.
- We've used a nominal 24V here, in practise the voltage will start higher [27.6V] and end lower [21V].

But this is a race - I wanna go fast!

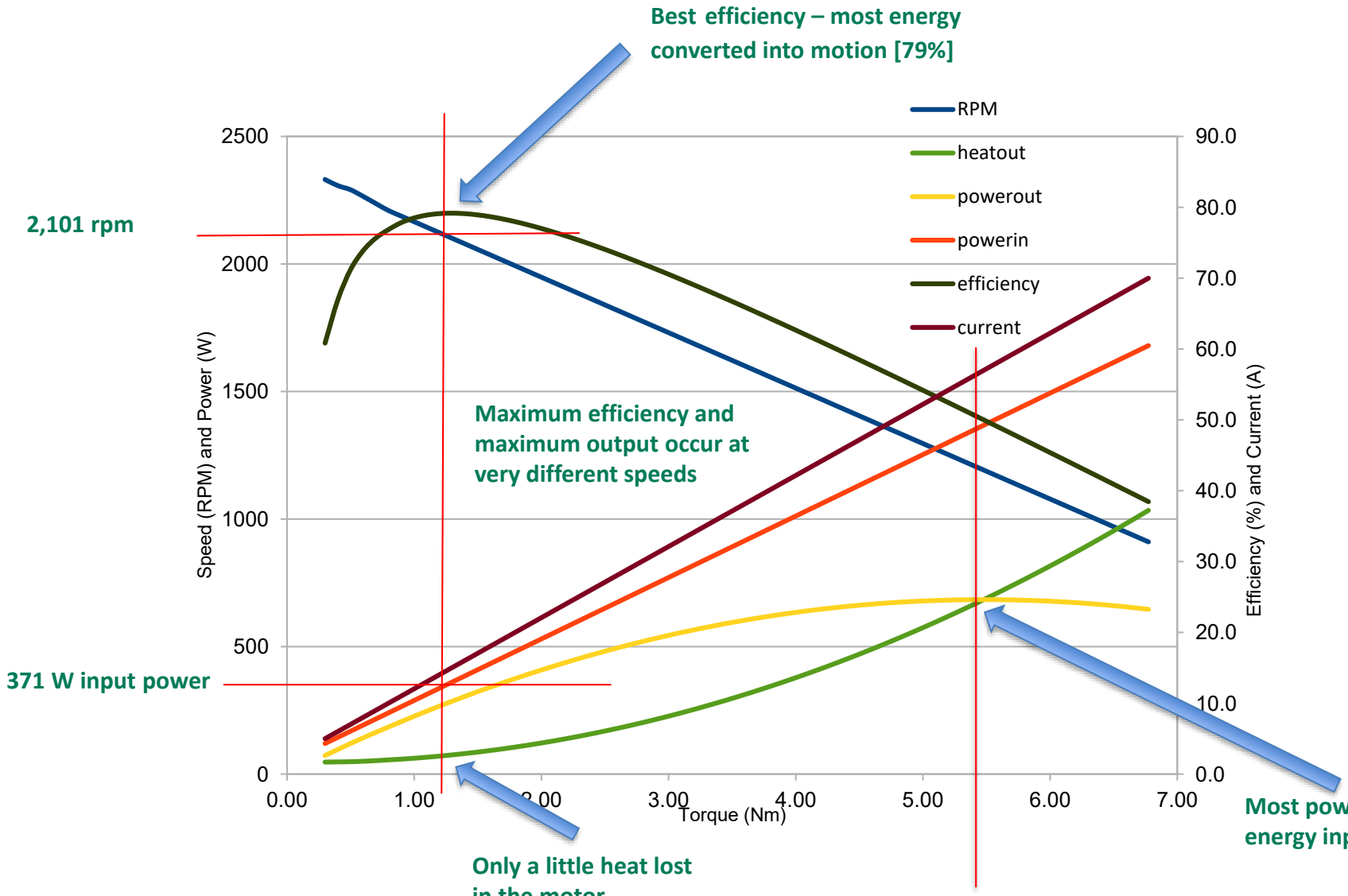
Sure buddy – be my guest, but just look at this chart…..



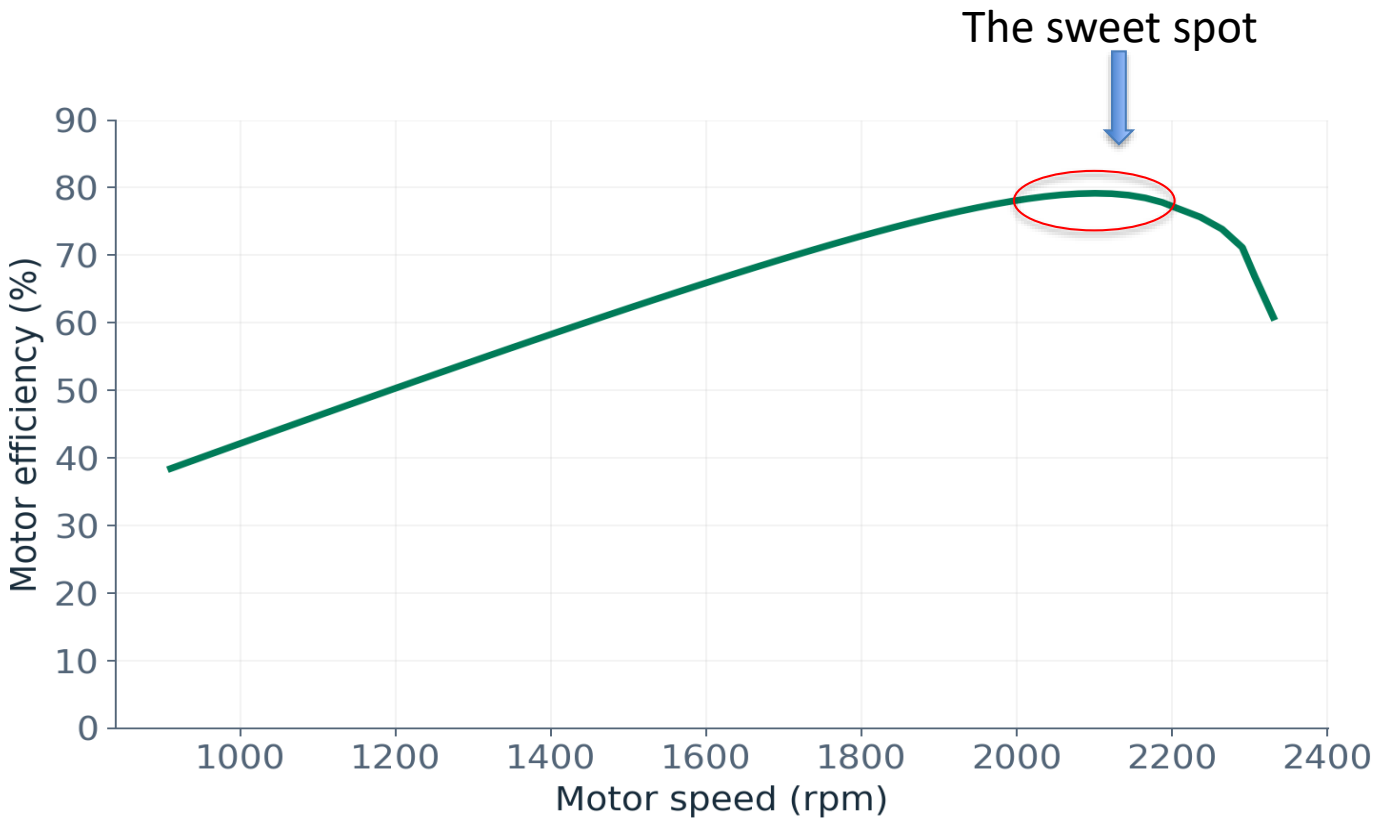
The winning number is distance after 90 minutes — not maximum speed.

Match your pace and gearing to the full-race energy budget.

What the motor performance curves tells us



Looked at another way….



Which funnily enough is pretty close to what the batteries will provide

2,101 rpm

15 A

About 77–79% efficiency

Around 2,000–2,200 rpm

Go outside of this RPM range and you start wasting energy as heat instead of forward motion

Efficient rpm is a region under load — not a fixed target.

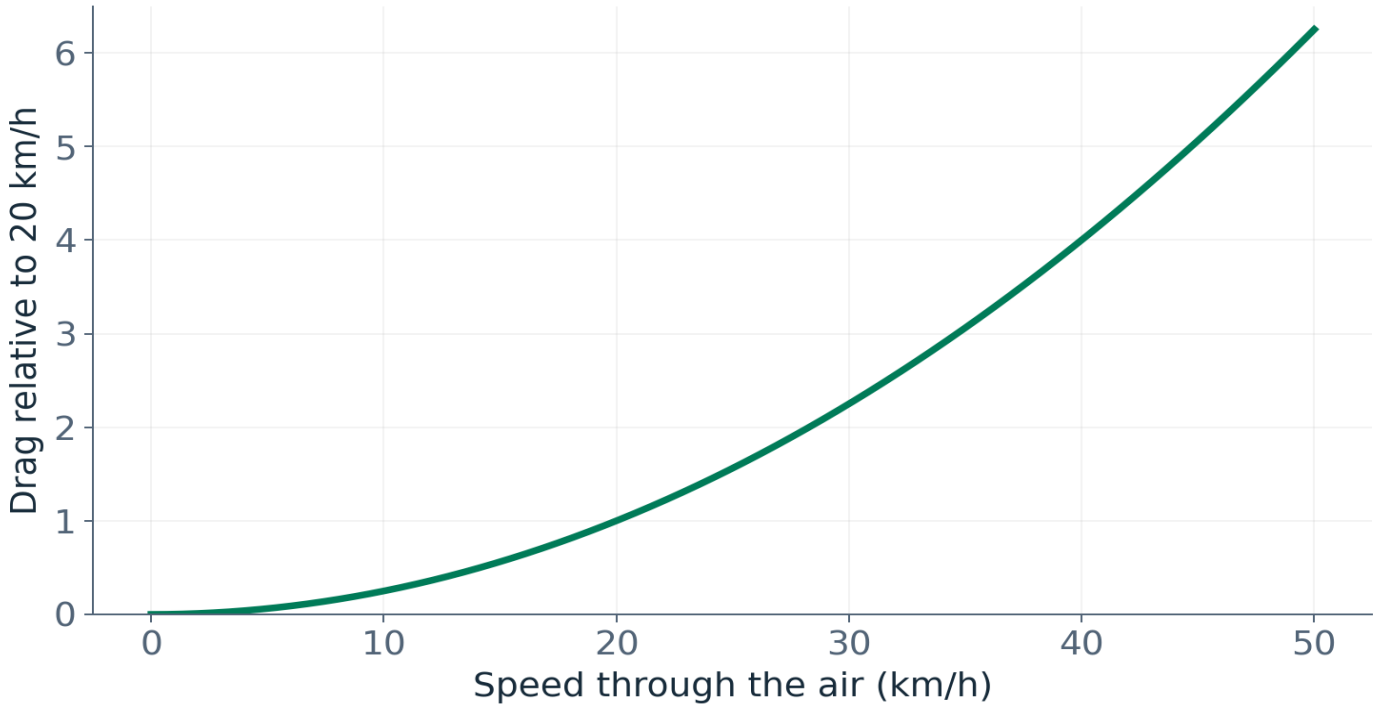
Source: supplied Greenpower Motor Performance Graph – Mar 18.xlsx; 24 V table.

So what about gearing…. why a tall gear ratio makes the motor work harder

Same 30 km/h road speed	Tall 5 : 1	Near target 6.6 : 1	Short 8 : 1
Motor rpm	1,592	2,101	2,546
Required shaft torque	1.73 Nm	1.31 Nm	1.08 Nm
Estimated motor current	19.3 A	15.1 A	12.8 A*

Tall gearing requires more torque and therefore more current.

Air resistance rises with speed squared



20 → 30 km/h

1.5× speed

2.25× drag

20 → 40 km/h

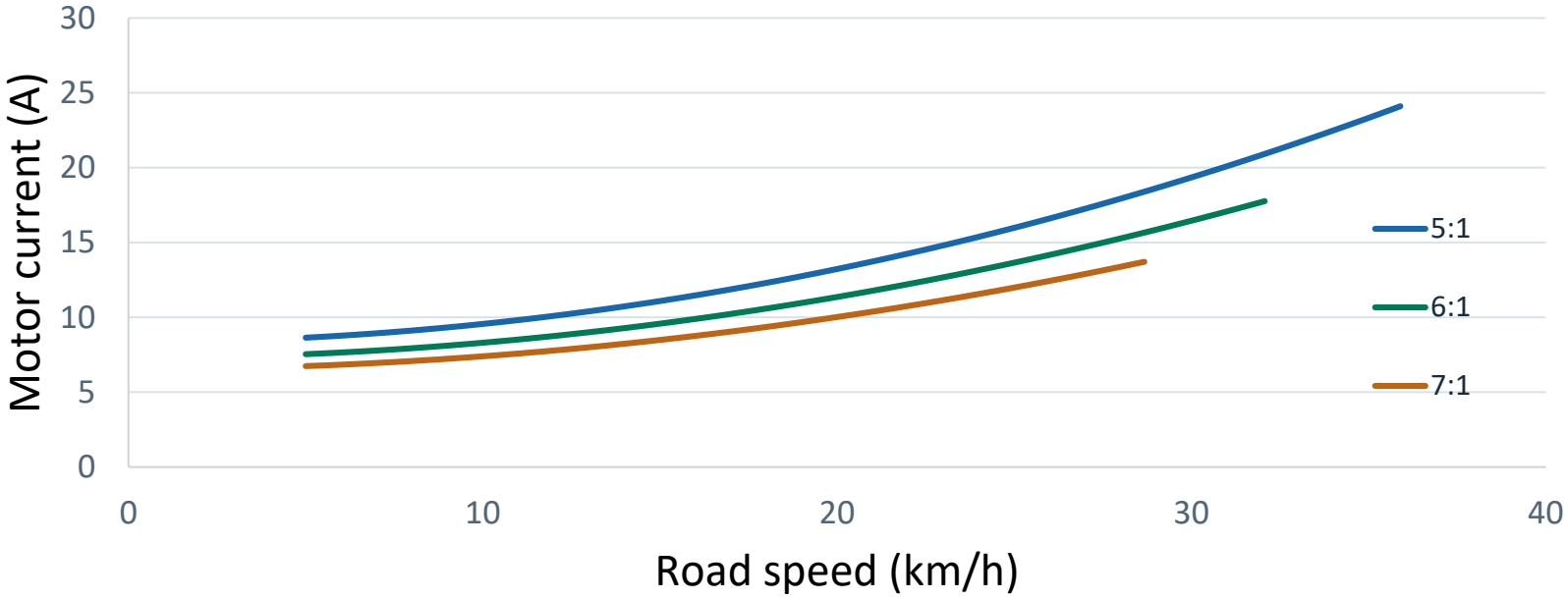
2× speed

4× drag

Drag force: F = ½ × air density × CdA × air speed²

Source: NASA Glenn, Drag Equation.

Motor current vs road speed



At the same road speed, 5:1 needs the most motor current.

Motor current rises roughly with speed²

More speed increases air resistance and therefore the torque needed

PWM battery current differs from motor current. Curves estimate steady running, excluding acceleration and hills.

Model: 0.50 m wheel, CdA 0.50 m², rolling force 12 N, chain efficiency 95%. Torque/current: supplied motor table.

A 20% speed increase has a large energy cost

+20%

Speed through the air

+44%

Aerodynamic force

and energy per kilometre

+73%

Aerodynamic power

in still air

A small top-speed gain can consume a large part of your energy budget.

Re-run with the supplied motor performance

Strategy	Battery power	Powered time	Distance
30 km/h; matched ratio	364 W	90 min	45.0 km
40 km/h; matched ratio	838 W	43.0 min	28.6 km

Here, the 40 km/h design loses 16.4 km by exhausting its energy early.

Source: supplied Greenpower Motor Performance Graph – Mar 18.xlsx; 24 V table.

Smooth speed beats repeated sprints

Same average speed	Steady 30 km/h	20 / 40 km/h, equal time
Average speed	30 km/h	30 km/h
Relative aero power*	1.00	1.33
Additional effects	Smooth load	Acceleration and braking

i.e. 33% more aero losses

Avoid unnecessary speed peaks; drive safely and consistently through traffic.

*Mean aerodynamic power proportional to $(20^3+40^3)/2$ is 33.3% greater than 30^3. No additional acceleration / braking loss included.

Calculate a ratio from the real efficiency point

Step	Worked example
Choose sustainable road speed	30 km/h = 8.33 m/s
Measure rolling wheel size	0.50 m diameter → 1.57 m circumference
Calculate wheel speed	8.33 ÷ 1.57 × 60 = 318 rpm
Use listed efficient motor speed	2,101 rpm at 24 V, 15 A
Calculate reduction ratio	2,101 ÷ 318 = 6.60 : 1

Example: 79-tooth wheel sprocket ÷ 12-tooth motor sprocket = 6.58 : 1.

Turn the theory into a team test

- Log speed, motor rpm, battery voltage, battery current and temperature.
- Try ratios near the calculated starting point under comparable conditions.
- Compare Wh/km, lap consistency and motor heating.
- Repeat at lower battery voltage, including required driver changes.

Change one thing at a time. Judge the whole 90-minute result.

Five ideas to take back to the workshop

- Air resistance ∝ air speed² in still air.
- Choose the fastest sustainable average pace for the full race.
- This motor’s best listed efficiency is near 2,100 rpm at 24 V.
- Gear the car to reach the useful motor region under load.

Teacher notes: scope and sources

Motor curves: supplied March 2018 spreadsheet.

Road model: 600 Wh usable; CdA 0.50 m^2; 12 N rolling force; 95% chain efficiency.

Peak-efficiency rpm is a starting point, not a complete race strategy.

Motor data: Greenpower Motor Performance Graph – Mar 18.xlsx

Greenpower: current rules and bulletins

Greenpower: current motor listing

Greenpower: Formula 24 race format

NASA: drag equation

maxon: motor and back-EMF equations